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Mathematical Model for Dynamic Damage Probability of the Repetition Pulse Rate High-energy Laser System

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ABSTRACT

Aimed at the high-energy laser system, under the assumption that the tracking error is the normal stochastic process of mean square differentiability and ergodicity, the Series Expression of the dynamic damage probability was given. The example demonstrate some characteristics of the dynamic damage probability as follow. The system proposed indicates the quantitative relationship between dynamic damage probability and transfer function of the tracking error system, which offers theoretical and technological support on proofing, designing and testing the dynamic damage probability of laser system.

Keywords: high power laser system; tracking error; stochastic passage characteristic; damage probability

I. INTRODUCTION

With the advantages of high reliability, small size and high efficiency, the repetition rate pulsed laser system, which is a representative of the diode pumped solid state laser, has become an important direction of the development of high energy laser system. It can make the laser energy density in a small area within a relatively small area of the target surface in a short time by the dense emission of high power laser pulse. When a stationary target is directly irradiated with a stationary target, the different characteristics of the high energy laser in different environments, the damage ability of different materials, and the static damage characteristics of the high energy laser, are pointed. A lot of research literature about this characteristic, the literatures[1-5] on the theoretical analysis and a large number of reliable test results generalize a universal, the laser damage characteristics of static: in the uncertain environment of direct irradiation target specific material a certain time (or time of ruin causing damage caused by pulse number), can lead to the damage; if the conditions change is deterministic, causing ruin time changes induced by intense laser; destroy target time is very short, due to temperature rise caused destruction can be approximated a thermal process, namely in a limited time, causing ruin time can be decomposed into multiple periods followed by irradiation, as long as the irradiation time, the damage effect will remain unchanged. In this paper, the dynamic damage probability is derived, and the characteristics of the dynamic damage probability are analyzed by a numerical example.

II. PROBLEM DESCRIPTION

When using repetition rate pulsed laser device irradiates a target, it is reasonable to use circular area irradiated as the shooting gate. Then it is obviously that the shooting progress is a discrete time series composed of several pulses. The cross situation of direction tracking error to shooting gate with radius ρ is shown in Fig.1.

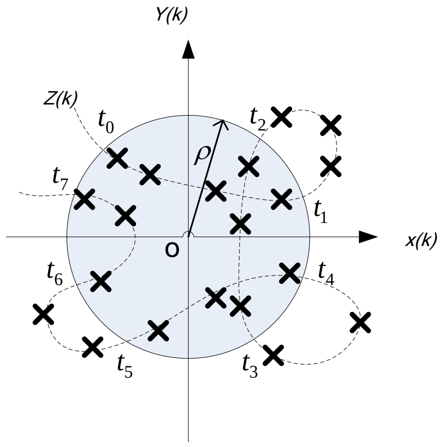

Fig 1.the tracking error randomly crosses the shooting gate

Where the symbols "×" marked in black are samples of the laser pulses. Time series composed of samples within the circuit area is the irradiation period, moreover, samples out of circuit area compose the non-irradiation period . The two periods alternately appear during the whole shooting process which is a normal stationary ergodic Markov process with discrete parameter[6].

To dynamically destroy or injury a target, laser system should match 3 damage conditions as below :

1> The total number of irradiation pulses when laser beam covers a fixed circular area (the point O is the center in the Fig 1.) on the target exceeds the Damage Threshold(T_h), then the target can be judged to be destroyed.

2> The tracking error of laser system,

Assume that $Z(k)=\left(x(k),y(k)\right)^T$, is a mutually independent, mean square differentiability, ergodicity two dimensional Gaussian process, whose mean value is zero.

3> The laser launches at $t=t_0$ and ends at $T_s > T_h > 0$, where T_s is the total number of laser pulses during a shooting process.

Once it matches the conditions listed above, when the quantity of pulses in one laser shoot process is fixed, the target will be damaged or injured while the total numbers of irradiation exceeds Damage Threshold, this paper mainly discussed the dynamic damage probability under such condition.

III. The Stochastic Passage Characteristics of the Tracking Error

The density function of $Z(k)$ can be marked respectively as follows[6]:

$$f[Z(k)] = f[x(k)]f[y(k)] = \frac{1}{\sqrt{2\pi}\sigma_x}\exp[-\frac{(x(k))^2}{2\sigma_x^2}] \frac{1}{\sqrt{2\pi}\sigma_y}\exp[-\frac{(y(k))^2}{2\sigma_y^2}] \quad (0.1)$$

Where σ_x^2 and σ_y^2 σ_y are the variances of $x(k)$ and $y(k)$.

As shown in Fig. 1, there are two situations in a stochastic passage period. One is that it starts from the irradiation point with probability of α_0, the other is that it starts from non-irradiation point with probability of α_1 [7],they can defined as

$$\alpha_0(Z) = \iint\limits_{x^2(k)+y^2(k)\le\rho} \frac{1}{2\pi\sigma_x\sigma_y}\exp [-\frac{(x(k))^2}{2\sigma_x^2} - \frac{(y(k))^2}{2\sigma_y^2}]\mathrm{d}x(k)\mathrm{d}y(k) \quad (0.2)$$

$\alpha_1(Z) = 1-\alpha_0(Z)$ if $\sigma_x = \sigma_y$, then

$$\alpha_0(Z) = 1-\exp\left(-\frac{\rho^2}{2\sigma_x^2}\right) \quad (0.3)$$

Then, the probability transition matrix is[7]

$$\boldsymbol{P}(Z) = \begin{pmatrix} P_{00}(Z) & P_{01}(Z) \\ P_{10}(Z) & P_{11}(Z) \end{pmatrix}$$

The first element in the matrix is defined as

$$P_{00}(Z) = P\{Z(k+1) \in \Omega \mid Z(k) \in \Omega\}$$
$$= \frac{1}{\alpha_0(Z)} \iint_{x^2(k)+y^2(k)\le\rho} \iint_{x^2(k)+y^2(k)\le\rho} f[Z(k+1) \mid Z(k)] f[Z(k)] \mathrm{d}Z(k+1)\mathrm{d}Z(k)$$
$$= \frac{1}{\alpha_0(Z)} \iint_{x^2(k)+y^2(k)\le\rho} \iint_{x^2(k)+y^2(k)\le\rho} \frac{1}{4\pi^2\sigma_x^2\sigma_y^2\sqrt{(1-r_x^2)(1-r_y^2)}} \exp[-\frac{(x(k))^2}{2\sigma_x^2} - \frac{(x(k+1)-r_x x(k))^2}{2\sigma_x^2(1-r_x^2)} - \frac{(y(k))^2}{2\sigma_y^2} - \frac{(y(k+1)-r_y y(k))^2}{2\sigma_y^2(1-r_y^2)}] \mathrm{d}x(k+1)\mathrm{d}y(k+1)\mathrm{d}x(k)\mathrm{d}y(k) \quad (0.4)$$

Where r_x and r_y represent the correlation coefficients of the adjacent two sampling points for $x(k)$ and $y(k)$ respectively. The other elements can also be calculated uniformly.

IV. THE DISTRIBUTIONS FUNCTION OF IRRADIATION PERIOD AND NON-IRRADIATION PERIOD

Consider $Z(k)$ to be a zero mean, ergodic stationary random Gaussian process, let Ω be the area of shooting gate, the distribution of $Z(k)$ in and outside the shooting gate is illustrated in Fig.2.

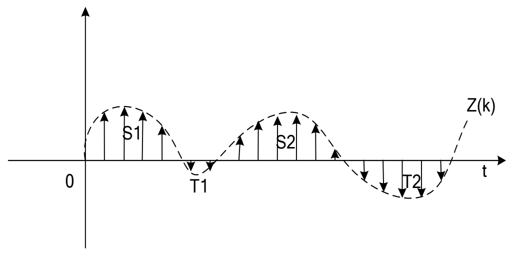

Fig 2 $Z(k)$ showing as irradiation period and non-irradiation period

From Fig 2., the irradiation time series are defined as $S_i = \{S_1, S_2, \ldots i = 1, 2, \ldots\}$,which are positive, independent, identically , distributed, random variables with distribution function $F(x)$; the irradiation time series are defined as $T_i = \{T_1, T_2, \ldots i = 1, 2, \ldots\}$, which are positive, independent, identically , distributed, random variables with distribution function $G(x)$ .there is defined as

$$S_0 = 0, \quad S_n = \sum_{i=1}^{n} S_i, \quad n \ge 1$$

and

$$T_n = \sum_{i=1}^{n} T_i, \quad n \ge 1$$

The non-irradiation period obeys the geometric distribution [7]

$$F(x=k) = \boldsymbol{P}\{T_i = kT \mid Z(k) \notin \Omega\} = P_{11}^{k-1}(Z) P_{10}(Z) \quad (0.5)$$

The irradiation period also obeys the geometric distribution

$$G(x=k) = P\{S_i = kT \mid Z(k) \in \Omega\} = P_{00}^{k-1}(Z) P_{01}(Z) \quad (0.6)$$

Where $k \ge 1$, T is the sample time。

From formula (0.6), the distribution of n irradiation periods is

$$F_n = \frac{(n+k-1)!}{k!(n-1)!} P_{00}^{k-1} P_{01}^{n} \quad (0.7)$$

From formula (0.5), the distribution of n non-irradiation periods is

$$G_n = \frac{(n+k-1)!}{k!(n-1)!} P_{10}^{n-1} P_{11}^{k-1} \quad (0.8)$$

V. THE DYNAMIC DAMAGE PROBABILITY OF THE LASER SYSTEM

Because the shooting moment of laser system start at t_0 (Fig 1), so the first period is irradiation period, then the stochastic period will be renewal alternatively between irradiation period and non-irradiation period. Since every period is randomized, any random period is possibly ended at the time when stops shooting.

If the pulse counts of the laser belonged to the irradiation periods beyond T_h, the target will be damaged. Assume T_s is the total pulse counts of the laser in once shooting process, then the dynamic damage probability of the laser system can be calculated as follow.

Case 1: if the target is damaged in the first irradiation period, the probability is

$$H_0(x = T_h T_m) = 1 - P_{00}^{T_h - 1}(Z) P_{01}(Z) \qquad (0.9)$$

Case 2:if the target is damaged needing more than one irradiation period, the probability is

$$H_n = \sum_{n=2}^{\infty} \{ P(T_n \le T_s - T_h) \cdot \left[P(S_n \le T_h) - P(S_{n+1} \le T_h) \right] \} \qquad (0.10)$$

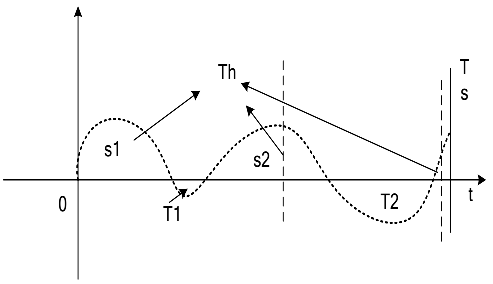

Fig 3.The damage case experiencing multiple irradiation periods

Proof From Fig 3, the dashed vertical lines indicate the possible time when the irradiation pulse number has reached the Damage Threshold(T_h).Let $N(t)$ be the renewal process generated by $S_1, S_2 \ldots$. Using a renewal process property, it follows instantly that

$$\begin{aligned} &P\{N(t) = n\} = \\ &P\{N(t) \ge n\} - P\{N(t) \ge n+1\} \\ &= F_n(t) - F_{n+1}(t), \quad t \ge 0, \quad n = 1, 2, \ldots \end{aligned} \qquad (0.11)$$

In once shooting process, the total laser pulse number is T_s, the probability of the irradiation pulse number more tan T_h equivalent to the probability of the irradiation pulse number less than $T_s - T_h$:

$$P(S_n \ge T_h) = P(T_n \le T_s - T_h) \qquad (0.12)$$

So the probability is

$$H_n = \sum_{n=2}^{\infty} P(T_n \le (T_s - T_h)) \left[P(S_n \le T_h) - P(S_{n+1} \le T_h) \right] \qquad (0.13)$$

Taking into account that the total laser pulse number is limited by T_s,the Maximum value of $N(t)$ is

$$N(t) = \min\{T_h, T_s - T_h\} \qquad (0.14)$$

From the two cases, the total dynamic damage probability of the laser system is

$$\begin{aligned} P &= 1 - (P_{00})^{T_h - 1}(1 - P_{00}) \\ &+ \sum_{n=2}^{N(t)} \left\{ \left(\sum_{k=1}^{T_s - T_h} \frac{(n+k-1)!}{k!(n-1)!} (P_{11})^{k-1} (1 - P_{11})^n \right) \cdot \right. \\ &\left[\sum_{k=1}^{N(t)} \frac{(n+k-1)!}{k!(n-1)!} P_{00}{}^{k-1} (1 - P_{00})^n \right. \\ &\left. \left. - \sum_{k=1}^{N(t)} \frac{(n+k)!}{k!n!} P_{00}{}^{k-1} (1 - P_{00})^{n+1} \right] \right\} \end{aligned} \qquad (0.15)$$

VI. SIMULATION RESULT

Assumed the laser system emission frequency is fr, the damage threshold of the target: $T_h = 150$, the laser emission time is 5 seconds, $P_{01} = 1 - P_{00} = 0.2$, $P_{10} = 1 - P_{11} = 0.3$,the relationship between the fr and the dynamic damage probability is shown below:

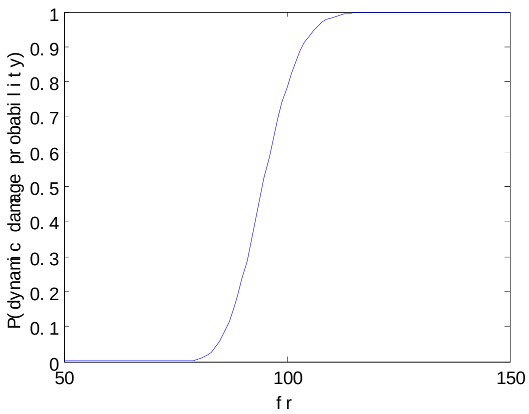

Fig 5 Relationship between *fr* and the damage probability

From Fig 5 we can see that in one shoot process, the total number of laser pulse needs to exceed the damage threshold, will achieve the goal of the damage, that is, the laser pulse frequency has a minimum requirements; greater than the lowest value of the laser emission frequency and damage probability is now a monotonic increasing relationship.

VII. Conclusion

Because of the support of stochastic thesis during formulate deductions, it is reliable when laser system satisfies the conditions proposed in this paper, the conclusions given are inerrant. Beyond that, since original data required in calculating the dynamic damage probability contains the tracking error and relevant variances of the derivative of the laser system, it can either be seen as an evidence of the probability, or directly combined with the transfer function or state equation of laser system to optimize control strategy, while treat the probability as the objective function.

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